The NASA Langley Challenge on Optimization Under Uncertainty

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Abstract

This paper presents an Uncertainty Quantification (UQ) challenge focusing on key aspects of model calibration, sensitivity analysis, uncertainty reduction, and reliability-based design in the presence of aleatory and epistemic uncertainty.

Keywords: Model calibration, epistemic uncertainty, robust optimization, risk, sensitivity analysis.

1. Introduction

NASA missions often involve the development of new vehicles and systems that must be designed to operate in harsh domains with a wide array of operating conditions. These missions involve high-consequence and safety-critical systems for which quantitative data is either very sparse or prohibitively expensive to collect. Limited heritage data may exist, but is also usually sparse and may not be directly applicable to the system of interest, making UQ extremely challenging. NASA modeling and simulation standards require estimates of uncertainty and descriptions of any processes used to obtain these estimates. The NASA Langley Research Center has developed a UQ challenge problem in an effort to focus a community of researchers towards common goals. While the problem formulation is written in a discipline-independent framework, the underlying application is consistent with the complexities of realistic systems.

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The dynamic system at the core of this challenge problem is highly relevant to a wide variety of systems faced by the dynamics systems and control communities. The key attributes of this challenge problem were chosen to be representative of the analysis and design tasks required to model and control flexible structures subject to uncertainty. Concrete applications embodying this framework are aircraft gust alleviation, aeroelastic control, flutter suppression, and spacecraft precision pointing. Key features of underlying system are: a computational model identified and validated using limited data, uncertain models parameters and boundary conditions resulting from poorly-known subsystem interconnections, and the need to suppress undesirable oscillations of a flexible structure by means of a feedback controller with a non-collocated sensor/actuator pair.

The computational model and relevant data can be downloaded from: *https://uqtools.larc.nasa.gov/nasa-uq-challenge-problem-2020/*. Each participating team must first register using this website.

2. Uncertainty Classification

This challenge problem adopts the generally accepted classification of uncertainty referred to as aleatory and epistemic [3], [4], [1]. *Aleatory* uncertainty (also called irreducible or stochastic) is caused by inherent variation or randomness. As such, aleatoric parameters are often modeled as random variables. In contrast, *epistemic* uncertainty is caused by lack of knowledge in the true value of a parameter. Therefore, epistemic uncertainty is not an inherent property of the system, but instead it represents the state of knowledge of the analyst. In the context of this challenge, an epistemic variable can take on any fixed value within a bounded set. According to its physical origin, the value of a parameter can be either fixed (e.g., the mass of a specific element produced by a manufacturing process) or varying (e.g., the mass of any element that can be produced by a manufacturing process). The physical origin of a parameter as well as the knowledge we have about it must be used to create an Uncertainty Model (UM) for it. Intervals, fuzzy sets, random variables, probability boxes (a.k.a. pboxes) [2], etc., are commonly used classes of UMs.

Because most models, especially those characterizing uncertainty, are imperfect; the possibility of improving them always exists. A reduction of the uncertainty in an epistemic variable is attained by reducing the size of the set where the true value of such a variable is expected to be. This reduction can be attained by performing additional experiments or doing better computational simulations.

3. Framework

A computational model of a physical system will be used to evaluate and improve its reliability. Denote by $\delta \in \mathbb{R}^{n_\delta}$ a parameter of the model whose value is uncertain, and by $\theta \in \mathbb{R}^{n_\theta}$ a design variable to be prescribed by the analyst. The parameter δ is comprised of elements of a and e, where $a \in \mathbb{R}^{n_a}$ and $e \in \mathbb{R}^{n_e}$ are aleatory and epistemic variables respectively. The UM for a will be denoted as $a \sim f_a$, where[1] f_a is a joint density supported in the set A. In contrast, the UM for e will be denoted as $e \sim E$, where E is a hyper-rectangular set. Hence, the UM of δ is fully prescribed by the pair $\langle f_a, E \rangle$. In this challenge the functional form of f_a and the center and diagonal of E are to be chosen by the respondents. A variable that depends on both epistemic and aleatory variables is fully characterized by a pbox.[2]

The system of interest is modeled as a set of interconnected subsystems. However, the uncertain parameter δ is concentrated onto a single subsystem. This subsystem is modeled by the function $y(a, e, t)$, where $y : \mathbb{R}^{n_a} \times \mathbb{R}^{n_e} \times [0, T] \to \mathbb{R}$ and t is time. The integrated system is modeled by $z\,(a, e, \theta, t)$, where $z : \mathbb{R}^{n_a} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_\theta} \times [0, T] \to \mathbb{R}^2$. Hence, the output of the subsystem is a function of time, whereas the output of the integrated system are two

[1] The probability density function, and the cumulative distribution function of u with parameter p will be denoted as $f_u(u; p)$ and $F_u(u; p)$ respectively.

[2] Consider $u(a, e, \theta)$, where $u : \mathbb{R}^{n_a} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_\theta} \to \mathbb{R}$, $a \sim f_a$, $e \sim E$ and θ is a parameter. When θ is fixed, u is fully prescribed by the pbox $B_u(u; \theta)$. This pbox is obtained by associating to each u the set of CDF values $F_u(u; e)$ corresponding to all values of e in E, i.e., $B_u(u; \theta) = \{F_u = F_u(u; e), \forall e \in E\}$. This interval-valued function can be written as $B_u(u; \theta) = \left[\underline{F}_u(u), \overline{F}_u(u)\right]$, where the CDFs $\underline{F}_u$ and $\overline{F}_u$ are the lower and upper pbox boundaries respectively. Each member of the family of infinitely many CDFs lies between them, and no tighter containing functions exist. The pbox boundaries are not pbox members in general. An inner approximation to B_u can be calculated from n samples of a and m samples of e by using

$$B_u^{n,m}(u; \theta) = \frac{1}{n} \left[\min_{i=1,\ldots m} \sum_{j=1}^{n} \mathbb{1}_{u\left(a^{(j)}, e^{(i)}, \theta\right) \leq u}, \max_{i=1,\ldots m} \sum_{j=1}^{n} \mathbb{1}_{u\left(a^{(j)}, e^{(i)}, \theta\right) \leq u} \right] \subseteq B_u(u; \theta),$$

where $\mathbb{1}_{(\cdot)}$ is the indicator function.

functions of time. Each of these functions will be given as a discrete time history, e.g., $y(t) = [y(0), y(dt), \dots, y(n_t\, dt)]$ where $n_t\, dt = T$.

Respondents will be asked to find a design point θ that yields a fast-decaying response $z_1(t)$ while keeping $z_2(t)$ below a given threshold. These design objectives will be cast as a set of reliability requirements. These requirements are fully prescribed by the performance functions $g(a, e, \theta)$, where $g : \mathbb{R}^{n_a} \times \mathbb{R}^{n_e} \to \mathbb{R}^{n_g}$. The system will be regarded as requirement compliant when $g(a, e, \theta) < 0$. For fixed values of θ and e, the set of a points where $g < 0$ is called the *safe domain*, whereas its complement set is called the *failure domain*. The worst-case performance function, defined as

$$w(a, e, \theta) = \max_{i=1,\dots n_g} g_i(a, e, \theta), \tag{1}$$

enables defining the safe and failure domains in terms of a single inequality, i.e., the safe domain is given by the a points where $w(a, e, \theta) < 0$. When the values of e and θ are fixed, and $a \sim f_a$; the worst-case performance function is given by the random variable $w \sim f_w(w; e, \theta)$ and the failure probability is $1 - F_w(0; e, \theta)$. In contrast, when θ is fixed, $e \sim E$, and $a \sim f_a$, the worst-case performance function is given by the pbox $w \sim B_w(w; \theta)$ and the probability of failure varies in the $[1 - \overline{F}_w(0; \theta), 1 - \underline{F}_w(0; \theta)]$ range.

An overview of the main goals of this challenge is as follows:

- Create an UM of δ according to observations of the subsystem.
- Choose a limited number of epistemic variables to refine.
- Perform a reliability analysis of a given design point.
- Seek a design point θ with improved reliability.
- Improve the UM of δ and θ according to observations of the integrated system.
- Improve θ by accepting a small risk.

4. Problem Statement

The particular reliability requirements of interest are introduced next. $g_1(a, e, \theta) < 0$ is needed for the system to be stable, $g_2 < 0$ with

$$g_2 = \max_{t \in [T/2, T]} |z_1(a, e, \theta, t)| - 0.02, \tag{2}$$

for the settling time of z_1 to be sufficiently fast, and $g_3 < 0$ with

$$g_3 = \max_{t \in [0,T]} |z_2(a, e, \theta, t)| - 4, \quad (3)$$

for the energy consumption to be acceptable. These $n_g = 3$ requirements define competing design objectives: design points θ that contract the failure domain corresponding to one requirement might also expand the failure domain corresponding to another.

The challenge is divided into the six subproblems described below. The numerical setup is $n_a = 5$, $n_e = 4$, $n_\theta = 9$, $n_g = 3$, $T = 5$, $n_t = 5000$, $A_0 = [0, 2]^{n_a}$, $E_0 = [0, 2]^{n_e}$, $n_1 = 100$, $n_2 = 100$ and $\hat{r} = 0.05$. The dataset, the baseline design, the computational models, and means to evaluate g can be downloaded from the website provided below.

A. Model Calibration & UQ of the Subsystem

In this subproblem we seek to characterize the parameters of the subsystem according to a limited number of observations.

A1) Given the data sequence $D_1 = \{y^{(i)}(t)\}$ for $i = 1, \ldots n_1$, create an UM for δ such that $a \sim f_a$ for $a \in A \subseteq A_0$, and $e \sim E \subseteq E_0$.

A2) Explain the rationale that led you to choose a particular distribution class for a. Why is that distribution better than any other? Evaluate the degree of dependency among the parameters of the identified f_a. Explain the rationale that led you to chose the geometry of E. Evaluate the extent by which the identified UM underfits/overfits the data. How does the value of n_1 impact your answers?

B. Uncertainty Reduction

In this subproblem we seek to determine which epistemic variables are dominant.

B1) Rank the epistemic parameters according to their ability to improve the predictive ability of the computational model of the subsystem.

B2) Determine $0 \leq k \leq 4$ uncertainty reductions to make[3]. Each of such reductions is prescribed by an epistemic variable and a refinement type.

[3] Another opportunity for refinement will be available in Subproblem E. The total num-

Two refinement types are available: one focuses on lowering the upper limit of the bounding interval while the other one focuses on increasing the lower limit of the bounding interval[4]. Ask the NASA hosts for the refined UMs. Denote the reduced epistemic space E_1.

B3) Update the UM of δ and the parameter ranking such that $e \sim E \subseteq E_1$.

C. Reliability Analysis of Baseline Design

In this subproblem we seek to evaluate the reliability of a given design point θ_{baseline} according to the current UM.

C1) Evaluate the range of the failure probability for each individual requirement,

$$R_i(\theta) = \left[\min_{e \in E} \mathbb{P}[g_i(a, e, \theta) \geq 0], \max_{e \in E} \mathbb{P}[g_i(a, e, \theta) \geq 0]\right], \tag{4}$$

for $i = 1, \ldots n_g$, where $\mathbb{P}[\cdot]$ is the probability operator.

C2) Evaluate the range of the failure probability for all requirements

$$R(\theta) = \left[\min_{e \in E} \mathbb{P}\left[w(a, e, \theta) \geq 0\right], \max_{e \in E} \mathbb{P}\left[w(a, e, \theta) \geq 0\right]\right]. \tag{5}$$

C3) Rank the epistemic uncertainties according to the contraction of $R(\theta)$ that might result from their reduction.

C4) Identify representative realizations of $\delta \in A \times E$ having a comparatively large likelihood near the failure domain. Use these points to characterize qualitatively different transitions to failure. Show the corresponding time responses of the integrated system.

C5) Evaluate the severity of each individual requirement violation, as mea-

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ber of reductions a group can request shall not exceed 4. The same epistemic parameter can be refined multiple times.

[4] For example, the refinements $\{e_1^+\}$ and $\{e_3^-\}$ will focus on decreasing the upper limit of E in the e_1 dimension and increasing the lower limit of E in the e_3 dimension respectively.

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$$s_i(\theta) = \max_{e \in E} \mathbb{E}[g_i | g_i \geq 0] \mathbb{P}[g_i \geq 0], \tag{6}$$

for $i = 1, \ldots n_g$, where $\mathbb{E}[\cdot|\cdot]$ is the conditional expectation.

D. Reliability-Based Design

In this subproblem we seek to improve the system's reliability by identifying a new design point.

D1) Find a reliability-optimal design point θ_{new}. The respondents should choose a meaningful optimality criterion along with a computational viable approach to pursue θ_{new}.

D2) Perform the analysis of θ_{new} described in Subproblem C for the current UM.

E. Model Update and Design Tuning

In this subproblem we seek to improve the UM and the design by using observations of the integrated system corresponding to θ_{new}.

E1) Provide θ_{new} to the NASA hosts, who will give you the corresponding data sequence $D_2 = \{z^{(i)}(t)\}$ for $i = 1, \ldots n_2$.

E2) Use this sequence to update the UM and tune your design.

E3) Determine which $4-k$ refinements to make. Each refinement is prescribed by an epistemic variable and a refinement type (see B2). Ask the NASA hosts for the corresponding UMs. Denote the reduced epistemic space E_2.

E4) Update the UM of δ and further improve the design such that $e \sim E \subseteq E_2$. Denote the resulting design point θ_{final}.

E5) Perform the analysis of θ_{final} described in Subproblem C for the current UM.

E6) Compare θ_{baseline}, θ_{new} and θ_{final} using the metrics in (4), (5) and (6).

F. Risk-Based Design

In this subproblem we seek a design point that accounts for *most* of the remaining epistemic space. The portion to be neglected has $r\%$ of the volume of E, where $r \in [0, 100)$ is called the risk.

F1) Propose a metric to quantify the gain, ℓ, resulting from taking the risk $r = \hat{r}$.

F2) Find a design point that maximizes $\ell(\hat{r})$ and denote it as $\theta_{\hat{r}\%\text{risk}}$. Explain the process used to choose the portion of E being ignored.

F3) Compare θ_{final} and $\theta_{\hat{r}\%\text{risk}}$ using the figures of merit above. Is it worth taking the $\hat{r}$ risk?

F4) Evaluate $\ell(r, \theta_{\text{final}})$ and $\ell(r, \theta_{\hat{r}\%\text{risk}})$ for a few values in $r \in [0, 10]$ to determine an acceptable level of risk (if any).

5. Software

The computational models of the subsystem and of the integrated system as well as the data will be given as MATLAB® files. These files, namely `yfun.m` (the subsystem), `zfun.m` (the integrated system), `gfun.m` (the performance functions), `baseline-design.mat` (the baseline design) and `D1.mat` (n observations of the response of the subsystem), can be downloaded from *https://uqtools.larc.nasa.gov/nasa-uq-challenge-problem-2020/*. The use of the functions, which require the Control Systems Toolbox to run, is exemplified in `test.m`.

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